5-2 Dividing Polynomials

Simplify.

1. $\frac{4xy^2 - 2xy + 2x^2y}{xy}$

SOLUTION:

$$\begin{aligned}\frac{4xy^2 - 2xy + 2x^2y}{xy} &= \frac{4xy^2}{xy} - \frac{2xy}{xy} + \frac{2x^2y}{xy} && \text{Sum of quotients}\\ &= 4y - 2 + 2x && \text{Simplify.}\\ &= 4y + 2x - 2 && \text{Simplify.}\end{aligned}$$

2. $(3a^2b - 6ab + 5ab^2)(ab)^{-1}$

SOLUTION:

$$\begin{aligned}\frac{3a^2b - 6ab + 5ab^2}{ab} &= \frac{3a^2b}{ab} - \frac{6ab}{ab} + \frac{5ab^2}{ab} && \text{Sum of quotients}\\ &= 3a - 6 + 5b && \text{Simplify.}\\ &= 3a + 5b - 6 && \text{Simplify.}\end{aligned}$$

3. $(x^2 - 6x - 20) \div (x + 2)$

SOLUTION:

$$x+2 \overline{\smash{)}\, x^2 - 6x - 20} \quad \text{quotient: } x - 8$$

$\underline{(-)\,x^2 + 2x}$	Multiply divisor by x.
$-8x - 20$	Subtract and bring down the next term.
$\underline{(-)\,-8x - 16}$	Multiply the divisor by -8.
-4	Subtract.

$$(x^2 - 6x - 20) \div (x + 2) = x - 8 - \frac{4}{x+2}$$

4. $(2a^2 - 4a - 8) \div (a + 1)$

SOLUTION:

$$\begin{array}{r} 2a - 6 \\ a+1 \overline{\smash{)}\, 2a^2 - 4a - 8} \\ \underline{2a^2 + 2a} \\ -6a - 8 \\ \underline{-6a - 6} \\ -2 \end{array}$$

$$(2a^2 - 4a - 8) \div (a + 1) = 2a - 6 - \frac{2}{a+1}$$

5. $(3z^4 - 6z^3 - 9z^2 + 3z - 6) \div (z + 3)$

SOLUTION:

$$\begin{array}{r}
3z^3 - 15z^2 + 36z - 105 \\
z+3\overline{)3z^4 - 6z^3 - 9z^2 + 3z - 6} \\
\underline{3z^4 + 9z^3} \qquad\qquad\qquad\qquad \\
-15z^3 - 9z^2 \qquad\qquad\quad \\
\underline{-15z^3 - 45z^2} \qquad\qquad\quad \\
+36z^2 + 3z \qquad\quad \\
\underline{+36z^2 + 108z} \qquad \\
-105z - 6 \\
\underline{-105z - 315} \\
309
\end{array}$$

$$(3z^4 - 6z^3 - 9z^2 + 3z - 6) \div (z+3) = 3z^3 - 15z^2 + 36z - 105 + \frac{309}{z+3}$$

6. $(y^5 - 3y^2 - 20) \div (y - 2)$

SOLUTION:

$$\begin{array}{r}
y^4 + 2y^3 + 4y^2 + 5y + 10 \\
y-2\overline{)y^5 + 0y^4 + 0y^3 - 3y^2 + 0y - 20} \\
\underline{y^5 - 2y^4} \qquad\qquad\qquad\qquad\qquad \\
2y^4 + 0y^3 \qquad\qquad\qquad\quad \\
\underline{2y^4 - 4y^3} \qquad\qquad\qquad\quad \\
4y^3 - 3y^2 \qquad\qquad\quad \\
\underline{4y^3 - 8y^2} \qquad\qquad\quad \\
+5y^2 + 0y \qquad\quad \\
\underline{+5y^2 - 10y} \qquad\quad \\
10y - 20 \\
\underline{10y - 20} \\
0
\end{array}$$

$$(y^5 - 3y^2 - 20) \div (y - 2) = y^4 + 2y^3 + 4y^2 + 5y + 10$$

7. **MULTIPLE CHOICE** Which expression is equal to $(x^2 + 3x - 9)(4 - x)^{-1}$?

A. $-x-7+\frac{19}{4-x}$

B. $-x-7$

C. $x+7-\frac{19}{4-x}$

D. $-x-7-\frac{19}{4-x}$

SOLUTION:

First rewrite the divisor so the *x*-term is first. Then use long division.

$$\begin{array}{r} -x-7 \\ -x+4\overline{\smash{)}x^2+3x-9} \\ \underline{x^2-4x} \\ 7x-9 \\ \underline{7x-28} \\ +19 \end{array}$$

$$\left(x^2+3x-9\right)\left(4-x\right)^{-1} = -x-7+\frac{19}{4-x}$$

The correct choice is A.

Simplify.

8. $(10x^2 + 15x + 20) \div (5x + 5)$

SOLUTION:

$$\begin{array}{r} 2x+1 \\ 5x+5\overline{\smash{)}10x^2+15x+20} \\ \underline{10x^2+10x} \\ 5x+20 \\ \underline{5x+5} \\ 15 \end{array}$$

$$\begin{aligned} 10x^2+15x+20 &= 2x+1+\frac{15}{5x+5} \\ &= 2x+1+\frac{3}{x+1} \end{aligned}$$

9. $(18a^2 + 6a + 9) \div (3a - 2)$

SOLUTION:

$$\begin{array}{r}
6a+6 \\
3a-2\overline{)18a^2+6a+9} \\
\underline{18a^2-12a} \\
18a+9 \\
\underline{18a-12} \\
+21
\end{array}$$

$$18a^2+6a+9=6a+6+\frac{21}{3a-2}$$

10. $\frac{12b^2+23b+15}{3b+8}$

SOLUTION:

$$\begin{array}{r}
4b-3 \\
3b+8\overline{)12b^2+23b+15} \\
\underline{12b^2+32b} \\
-9b+15 \\
\underline{-9b-24} \\
39
\end{array}$$

$$12b^2+23b+15=4b-3+\frac{39}{3b+8}$$

11. $\frac{27y^2+27y-30}{9y-6}$

SOLUTION:

$$\begin{array}{r}
3y+5 \\
9y-6\overline{)27y^2+27y-30} \\
\underline{27y^2-18y} \\
45y-30 \\
\underline{45y-30} \\
0
\end{array}$$

$$\frac{27y^2+27y-30}{9y-6}=3y+5$$

Simplify

12. $\dfrac{24a^3b^2 - 16a^2b^3}{8ab}$

SOLUTION:

$$\frac{24a^3b^2 - 16a^2b^3}{8ab} = \frac{24a^3b^2}{8ab} - \frac{16a^2b^3}{8ab} \quad \text{Sum of quotients.}$$
$$= 3a^2b - 2ab^2 \quad \text{Divide.}$$

13. $\dfrac{5x^2y - 10xy + 15xy^2}{5xy}$

SOLUTION:

$$\frac{5x^2y - 10xy + 15xy^2}{5xy}$$
$$= \frac{5x^2y}{5xy} - \frac{10xy}{5xy} + \frac{15xy^2}{5xy} \quad \text{Sum of quotients}$$
$$= x - 2 + 3y \quad \text{Divide.}$$
$$= x + 3y - 2 \quad \text{Simplify.}$$

14. $\dfrac{7g^3h^2 + 3g^2h - 2gh^3}{gh}$

SOLUTION:

$$\frac{7g^3h^2 + 3g^2h - 2gh^3}{gh}$$
$$= \frac{7g^3h^2}{gh} + \frac{3g^2h}{gh} - \frac{2gh^3}{gh} \quad \text{Sum of quotients}$$
$$= 7g^2h + 3g - 2h^2 \quad \text{Divide.}$$

15. $\dfrac{4a^3b - 6ab + 2ab^2}{2ab}$

SOLUTION:

$$\frac{4a^3b - 6ab + 2ab^2}{2ab}$$
$$= \frac{4a^3b}{2ab} - \frac{6ab}{2ab} + \frac{2ab^2}{2ab} \quad \text{Sum of quotients}$$
$$= 2a^2 - 3 + b \quad \text{Divide.}$$
$$= 2a^2 + b - 3 \quad \text{Simplify.}$$

16. $\dfrac{16c^4d^4 - 24c^2d^2}{4c^2d^2}$

SOLUTION:

$$\frac{16c^4d^4 - 24c^2d^2}{4c^2d^2}$$

$$= \frac{16c^4d^4}{4c^2d^2} - \frac{24c^2d^2}{4c^2d^2} \quad \text{Sum of quotients}$$

$$= 4c^2d^2 - 6 \quad \text{Divide.}$$

17. $\dfrac{9n^3p^3 - 18n^2p^2 + 21n^2p^3}{3n^2p^2}$

SOLUTION:

$$\frac{9n^3p^3 - 18n^2p^2 + 21n^2p^3}{3n^2p^2}$$

$$= \frac{9n^3p^3}{3n^2p^2} - \frac{18n^2p^2}{3n^2p^2} + \frac{21n^2p^3}{3n^2p^2} \quad \text{Sum of quotients}$$

$$= 3np - 6 + 7p \quad \text{Divide.}$$

18. **ENERGY** Compact fluorescent light (CFL) bulbs reduce energy waste. The amount of energy waste that is reduced each day in a certain community can be estimated by $-b^2 + 8b$, where b is the number of bulbs. Divide by b to find the average amount of energy saved per CFL bulb.

SOLUTION:

The average amount of energy saved per CFL bulb is:

$$\left(-b^2 + 8b\right) \div b = \frac{-b^2 + 8b}{b}$$
$$= -b + 8$$

19. **BAKING** The number of cookies produced in a factory each day can be estimated by $-w^2 + 16w + 1000$, where w is the number of workers. Divide by w to find the average number of cookies produced per worker.

SOLUTION:

The average number of cookies produced per worker is:

$$\left(-w^2 + 16w + 1000\right) \div w = \frac{-w^2 + 16w + 1000}{w}$$
$$= -w + 16 + \frac{1000}{w}$$

Simplify.

20. $(a^2 - 8a - 26) \div (a + 2)$

SOLUTION:

$$\begin{array}{r|rrr} \underline{-2} & 1 & -8 & -26 \\ & & -2 & +20 \\ \hline & 1 & -10 & |-6 \end{array}$$

$$(a^2 - 8a - 26) \div (a + 2) = a - 10 - \frac{6}{a+2}$$

21. $(b^3 - 4b^2 + b - 2) \div (b + 1)$

SOLUTION:

$$\begin{array}{r|rrrr} \underline{-1} & 1 & -4 & +1 & -2 \\ & & -1 & +5 & -6 \\ \hline & 1 & -5 & +6 & |-8 \end{array}$$

$$(b^3 - 4b^2 + b - 2) \div (b + 1) = b^2 - 5b + 6 - \frac{8}{b+1}$$

22. $(z^4 - 3z^3 + 2z^2 - 4z + 4)(z - 1)^{-1}$

SOLUTION:

$$\begin{array}{r|rrrrr} \underline{+1} & 1 & -3 & +2 & -4 & 4 \\ & & 1 & -2 & 0 & -4 \\ \hline & 1 & -2 & 0 & -4 & |0 \end{array}$$

$$(z^4 - 3z^3 + 2z^2 - 4z + 4) \div (z - 1) = z^3 - 2z^2 - 4$$

23. $(x^5 - 4x^3 + 4x^2) \div (x - 4)$

SOLUTION:

$$\begin{array}{r|rrrrrr} \underline{+4} & 1 & 0 & -4 & 4 & 0 & 0 \\ & & 4 & 16 & 48 & 208 & 832 \\ \hline & 1 & 4 & 12 & 52 & 208 & |832 \end{array}$$

$$(x^5 - 4x^3 + 4x^2) \div (x - 4) = x^4 + 4x^3 + 12x^2 + 52x + 208 + \frac{832}{x-4}$$

24. $\frac{y^3 + 11y^2 - 10y + 6}{y + 2}$

SOLUTION:

$$\begin{array}{r|rrrr} \underline{-2} & 1 & +11 & -10 & 6 \\ & & -2 & -18 & 56 \\ \hline & 1 & +9 & -28 & |62 \end{array}$$

$$\frac{y^3 + 11y^2 - 10y + 6}{y + 2} = y^2 + 9y - 28 + \frac{62}{y+2}$$

5-2 Dividing Polynomials

25. $(g^4 - 3g^2 - 18) \div (g - 2)$

SOLUTION:

$$\begin{array}{r|rrrrr} \underline{+2} & 1 & 0 & -3 & 0 & -18 \\ & & 2 & 4 & 2 & 4 \\ \hline & 1 & 2 & 1 & 2 & |-14 \end{array}$$

$$(g^4 - 3g^2 - 18) \div (g - 2) = g^3 + 2g^2 + g + 2 - \frac{14}{g-2}$$

26. $(6a^2 - 3a + 9) \div (3a - 2)$

SOLUTION:

$$\begin{array}{r} 2a + \frac{1}{3} \\ 3a - 2\overline{)6a^2 - 3a + 9} \\ \underline{6a^2 - 4a} \\ a + 9 \\ \underline{a - \frac{2}{3}} \\ \frac{29}{3} \end{array}$$

$$\begin{aligned}(6a^2 - 3a + 9) \div (3a - 2) &= 2a + \frac{1}{3} + \frac{\frac{29}{3}}{3a - 2} \\ &= 2a + \frac{1}{3} + \frac{29}{9a - 6}\end{aligned}$$

27. $\dfrac{6x^5+5x^4+x^3-3x^2+x}{3x+1}$

SOLUTION:

$$
\begin{array}{r}
2x^4+x^3-x+\frac{2}{3} \\
3x+1\overline{)\,6x^5+5x^4+x^3-3x^2+x} \\
\underline{6x^5+2x^4} \\
3x^4+x^3 \\
\underline{3x^4+x^3} \\
-3x^2+x \\
\underline{-3x^2-x} \\
2x+0 \\
\underline{2x+\frac{2}{3}} \\
-\frac{2}{3}
\end{array}
$$

$$(6x^5+5x^4+x^3-3x^2+x)\div(3x+1)=2x^4+x^3-x+\frac{2}{3}-\frac{2}{9x+3}$$

28. $\dfrac{4g^4-6g^3+3g^2-g+12}{4g-4}$

SOLUTION:

$$
\begin{array}{r}
g^3-\frac{1}{2}g^2+\frac{1}{4}g \\
4g-4\overline{)\,4g^4-6g^3+3g^2-g+12} \\
\underline{4g^4-4g^3} \\
-2g^3+3g^2 \\
\underline{-2g^3+2g^2} \\
g^2-g \\
\underline{g^2-g} \\
12
\end{array}
$$

$$(4g^4-6g^3+3g^2-g+12)\div(4g-4)=g^3-\frac{1}{2}g^2+\frac{1}{4}g+\frac{3}{g-1}$$

5-2 Dividing Polynomials

29. $(2b^3 - 6b^2 + 8b) \div (2b + 2)$

SOLUTION:

$$\frac{2b^3 - 6b^2 + 8b}{2b+2} = \frac{2(b^3 - 3b^2 + 4b)}{2(b+1)}$$
$$= \frac{b^3 - 3b^2 + 4b}{b+1}$$

Synthetic division:

$$\begin{array}{r|rrrr} \underline{-1} & 1 & -3 & +4 & 0 \\ & & -1 & +4 & -8 \\ \hline & 1 & -4 & +8 & |-8 \end{array}$$

$$\frac{b^3 - 3b^2 + 4b}{b+1} = b^2 - 4b + 8 - \frac{8}{b+1}$$

30. $(6z^6 + 3z^4 - 9z^2)(3z - 6)^{-1}$

SOLUTION:

$$\frac{6z^6 + 3z^4 - 9z^2}{3z-6} = \frac{3(2z^6 + z^4 - 3z^2)}{3(z-2)} = \frac{2z^6 + z^4 - 3z^2}{z-2}$$

Synthetic division:

$$\begin{array}{r|rrrrrrr} \underline{+2} & 2 & 0 & 1 & 0 & -3 & 0 & 0 \\ & & 4 & 8 & 18 & 36 & 66 & 132 \\ \hline & 2 & 4 & 9 & 18 & 33 & 66 & |132 \end{array}$$

$$\frac{2z^6 + z^4 - 3z^2}{z-2} = 2z^5 + 4z^4 + 9z^3 + 18z^2 + 33z + 66 + \frac{132}{z-2}$$

31. $(10y^6 + 5y^5 + 10y^3 - 20y - 15)(5y + 5)^{-1}$

SOLUTION:

$$\frac{10y^6+5y^5+10y^3-20y-15}{5y+5}$$

$$=\frac{5(2y^6+y^5+2y^3-4y-3)}{5(y+1)} \quad \text{Factor out GCF.}$$

$$=\frac{2y^6+y^5+2y^3-4y-3}{y+1} \quad \text{Simplify.}$$

$$\begin{array}{r|rrrrrrr} \underline{-1} & 2 & 1 & 0 & 2 & 0 & -4 & -3 \\ & & -2 & 1 & -1 & -1 & 1 & 3 \\ \hline & 2 & -1 & 1 & 1 & -1 & -3 & |0 \end{array}$$

$$\frac{2y^6+y^5+2y^3-4y-3}{y+1} = 2y^5 - y^4 + y^3 + y^2 - y - 3$$

32. **CCSS REASONING** A rectangular box for a new product is designed in such a way that the three dimensions always have a particular relationship defined by the variable x. The volume of the box can be written as $6x^3 + 31x^2 + 53x + 30$, and the height is always $x + 2$. What are the width and length of the box?

SOLUTION:

Divide the function by the height $(x + 2)$ to find the length and width. Use synthetic division.

$$\begin{array}{r|rrrr} \underline{-2} & 6 & 31 & 53 & 30 \\ & & -12 & -38 & -30 \\ \hline & 6 & 19 & 15 & |0 \end{array}$$

The depressed polynomial is $6x^2 + 19x + 15$.

$$\begin{aligned} 6x^3 + 31x^2 + 53x + 30 &= (x+2)(6x^2+19x+15) \\ &= (x+2)(2x+3)(3x+5) \end{aligned}$$

Since the volume of the box is the product of length, width and height, the width and length of box are $(2x + 3)(3x + 5)$.

33. **PHYSICS** The voltage V is related to current I and power P by the equation $V = \frac{P}{I}$. The power of a generator is modeled by $P(t) = t^3 + 9t^2 + 26t + 24$. If the current of the generator is $I = t + 4$, write an expression that represents the voltage.

SOLUTION:

$$V(t) = \frac{t^3 + 9t^2 + 26t + 24}{t + 4}$$

Synthetic division:

$$\begin{array}{r|rrrr} \underline{-4} & 1 & 9 & +26 & 24 \\ & & -4 & -20 & -24 \\ \hline & 1 & 5 & +6 & |0 \end{array}$$

$$V(t) = t^2 + 5t + 6$$

34. **ENTERTAINMENT** A magician gives these instructions to a volunteer.
• Choose a number and multiply it by 4.
• Then add the sum of your number and 15 to the product you found.
• Now divide by the sum of your number and 3.
a. What number will the volunteer always have at the end?
b. Explain the process you used to discover the answer.

SOLUTION:

a. Let x be the number.

$$\begin{aligned} \frac{4x + (x + 15)}{x + 3} &= \frac{5x + 15}{x + 3} \\ &= \frac{5(x + 3)}{x + 3} \\ &= 5 \end{aligned}$$

b. Sample answer: Let x be the number. Multiply the x by 4 to get $4x$. Then add x +15 to the product to get $5x + 15$. Divide the polynomial by $x + 3$. The quotient is 5.

35. **BUSINESS** The number of magazine subscriptions sold can be estimated by $n = \frac{3500a^2}{a^2 + 100}$,where a is the amount of money the company spent on advertising in hundreds of dollars and n is the number of subscriptions sold.

a. Perform the division indicated by $\frac{3500a^2}{a^2 + 100}$.

b. About how many subscriptions will be sold if $1500 is spent on advertising?

SOLUTION:

a.
$$\begin{array}{r} 3500 \\ a^2 + 100 \overline{\smash{\big)}\,3500a^2 \qquad\qquad} \\ \underline{3500a^2 + 350{,}000} \\ -350{,}000 \end{array}$$

$$\frac{3500a^2}{a^2 + 100} = 3500 - \frac{350{,}000}{a^2 + 100}$$

b. There are 15 hundreds in 1500. Substitute $a = 15$.

$$\begin{aligned} n &= \frac{3500a^2}{a^2 + 100} \\ &= \frac{3500(15)^2}{15^2 + 100} \\ &= \frac{3500(225)}{325} \\ &\approx 2423 \end{aligned}$$

Therefore, about 2423 subscriptions will be sold.

Simplify.

36. $(x^4 - y^4) \div (x - y)$

SOLUTION:

$$\begin{aligned} \frac{x^4 - y^4}{x - y} &= \frac{(x^2 - y^2)(x^2 + y^2)}{(x - y)} && \text{Factor.} \\ &= \frac{(x + y)(x - y)(x^2 + y^2)}{(x - y)} && \text{Factor.} \\ &= (x^2 + y^2)(x + y) && \text{Simplify.} \end{aligned}$$

5-2 Dividing Polynomials

37. $(28c^3d^2 - 21cd^2) \div (14cd)$

SOLUTION:

$$\frac{28c^3d^2 - 21cd^2}{14cd} = \frac{7cd(4c^2d - 3d)}{14cd} \quad \text{Factor out GCF.}$$

$$= \frac{4c^2d - 3d}{2} \quad \text{Simplify.}$$

38. $(a^3b^2 - a^2b + 2b)(-ab)^{-1}$

SOLUTION:

$$\frac{a^3b^2 - a^2b + 2b}{-ab} = \frac{ab\left(a^2b - a + \frac{2}{a}\right)}{-ab} \quad \text{Factor.}$$

$$= -a^2b + a - \frac{2}{a} \quad \text{Simplify.}$$

39. $\dfrac{n^3 + 3n^2 - 5n - 4}{n + 4}$

SOLUTION:

$$\begin{array}{r|rrrr} \underline{-4} & 1 & 3 & -5 & -4 \\ & & -4 & +4 & 4 \\ \hline & 1 & -1 & -1 & |0 \end{array}$$

$$\frac{n^3 + 3n^2 - 5n - 4}{n + 4} = n^2 - n - 1$$

40. $\dfrac{p^3 + 2p^2 - 7p - 21}{p + 3}$

SOLUTION:

$$\begin{array}{r|rrrr} \underline{-3} & 1 & 2 & -7 & -21 \\ & & -3 & +3 & 12 \\ \hline & 1 & -1 & -4 & |-9 \end{array}$$

$$\frac{p^3 + 2p^2 - 7p - 21}{p + 3} = p^2 - p - 4 - \frac{9}{p + 3}$$

41. $\dfrac{3z^5+5z^4+z+5}{z+2}$

SOLUTION:

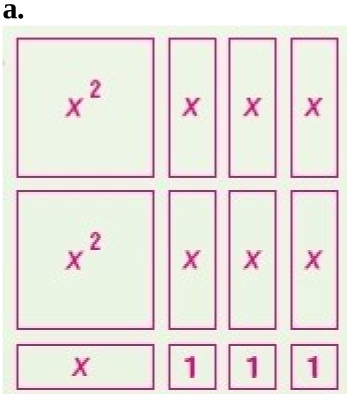
$$\begin{array}{r|rrrrrr} \underline{-2} & 3 & 5 & 0 & 0 & 1 & 5 \\ & & -6 & 2 & -4 & 8 & -18 \\ \hline & 3 & -1 & 2 & -4 & 9 & |-13 \end{array}$$

$$\frac{3z^5+5z^4+z+5}{z+2}=3z^4-z^3+2z^2-4z+9-\frac{13}{z+2}$$

42. **MULTIPLE REPRESENTATIONS** Consider a rectangle with area $2x^2 + 7x + 3$ and length $2x + 1$.
a. **CONCRETE** Use algebra tiles to represent this situation. Use the model to find the width.
b. **SYMBOLIC** Write an expression to represent the model.
c. **NUMERICAL** Solve this problem algebraically using synthetic or long division. Does your concrete model check with your algebraic model?

SOLUTION:

a.



The width is $x + 3$.

b. $(2x^2+7x+3)\div(2x+1)$

c.
$$\begin{array}{r} x+3 \\ 2x+1\overline{)2x^2+7x+3} \\ \underline{2x^2+x} \\ 6x+3 \\ \underline{6x+3} \\ 0 \end{array}$$

Yes, the concrete model checks with the algebraic model.

43. **ERROR ANALYSIS** Sharon and Jamal are dividing $2x^3 - 4x^2 + 3x - 1$ by $x - 3$. Sharon claims that the remainder is 26. Jamal argues that the remainder is –100. Is either of them correct? Explain your reasoning.

SOLUTION:

Use synthetic division.

$$\begin{array}{r|rrrr} \underline{3} & 2 & -4 & 3 & -1 \\ & & 6 & 6 & 27 \\ \hline & 2 & 2 & 9 & |26 \end{array}$$

The remainder is 26. So, Sharon is correct. Jamal actually divided by $x + 3$.

44. **CHALLENGE** If a polynomial is divided by a binomial and the remainder is 0, what does this tell you about the relationship between the binomial and the polynomial?

SOLUTION:

If a polynomial divided by a binomial has no remainder, then the polynomial has two factors: the binomial and the quotient.

45. **REASONING** Review any of the division problems in this lesson. What is the relationship between the degrees of the dividend, the divisor, and the quotient?

SOLUTION:

Sample answer: For example:

$$\begin{array}{r} 2a+\frac{1}{3} \\ 3a-2\overline{)6a^2-3a+9} \\ \underline{6a^2-4a} \\ a+9 \\ \underline{a-\frac{2}{3}} \\ \frac{29}{3} \end{array}$$

In this exercise, the degree of the dividend is 2. The degree of the divisor and the quotient are each 1. The degree of the quotient plus the degree of the divisor equals the degree of the dividend.

46. **OPEN ENDED** Write a quotient of two polynomials for which the remainder is 3.

SOLUTION:

Sample answer: Begin by multiplying two binomials such as $(x + 2)(x + 3)$ which simplifies to $x^2 + 5x + 6$. In order to get a remainder of 3 when divided, add 3 to the trinomial to get $x^2 + 5x + 9$. When divided, there will be a remainder of 3. The quotient of two polynomials is $\frac{x^2+5x+9}{x+2}$.

47. **CCSS ARGUMENTS** Identify the expression that does not belong with the other three. Explain your reasoning.

$3xy + 6x^2$	$\frac{5}{x^2}$
$x + 5$	$5b + 11c - 9ad^2$

SOLUTION:

$\frac{5}{x^2}$ does not belong with the other three. The other three expressions are polynomials. Since the denominator of $\frac{5}{x^2}$ contains a variable, it is not a polynomial.

48. **WRITING IN MATH** Use the information at the beginning of the lesson to write assembly instruction using the division of polynomials to make a paper cover for your textbook.

SOLUTION:

Sample answer: By dividing the area of the paper $140x^2 + 60x$ by the height of the book jacket $10x$, the quotient of $14x + 6$ provides the length of the book jacket. The front and back cover are each $6x$ units long and the spine is $2x$ units long. Then, subtracting $14x$, we are left with 6 inches. Half of this length is the width of each flap.

49. An office employs x women and 3 men. What is the ratio of the total number of employees to the number of women?

A $\frac{x+3}{x}$

B $\frac{x}{x+3}$

C $\frac{3}{x}$

D $\frac{x}{3}$

SOLUTION:

Total number of employees: $x + 3$
Number of women: x
Ratio of the total number of employees to the number of women is:

$$\frac{x+3}{x}$$

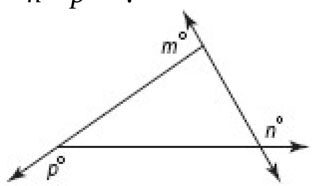
The correct choice is A.

50. **SAT/ACT** Which polynomial has degree 3?
A $x^3 + x^2 - 2x^4$
B $-2x^2 - 3x + 4$
C $3x - 3$
D $x^2 + x + 12^3$
E $1 + x + x^3$

SOLUTION:

To determine the degree of a polynomial, look at the term with the greatest exponent. $x^3 + x + 1$
The correct choice is E.

51. **GRIDDED RESPONSE** In the figure below,
$m + n + p = ?$



SOLUTION:

Sum of the exterior angles of a triangle is $360°$.
So, $m + n + p = 360°$

52. $(-4x^2 + 2x + 3) - 3(2x^2 - 5x + 1) =$
F $2x^2$
H $-10x^2 + 17x$
G $-10x^2$
J $2x^2 + 17x$

SOLUTION:

$$(-4x^2 + 2x + 3) - 3(2x^2 - 5x + 1)$$
$$= -4x^2 + 2x + 3 - 6x^2 + 15x - 3 \quad \text{Distributive Property}$$
$$= -10x^2 + 17x \quad \text{Combine like terms.}$$

The correct choice is H.

Simplify.

53. $(5x^3 + 2x^2 - 3x + 4) - (2x^3 - 4x)$

SOLUTION:

Use the Distributive Property and then combine like terms.

$$\begin{aligned}&(5x^3+2x^2-3x+4)-(2x^3-4x)\\&=5x^3+2x^2-3x+4-2x^3+4x\\&=3x^3+2x^2+x+4\end{aligned}$$

54. $(2y^3 - 3y + 8) + (3y^2 - 6y)$

SOLUTION:

$$\begin{aligned}(2y^3-3y+8)+(3y^2-6y)&=2y^3-3y+8+3y^2-6y\\&=2y^3+3y^2-9y+8\end{aligned}$$

55. $4a(2a - 3) + 3a(5a - 4)$

SOLUTION:

Use the Distributive Property and then combine like terms.

$$\begin{aligned}4a(2a-3)+3a(5a-4)&=8a^2-12a+15a^2-12a\\&=23a^2-24a\end{aligned}$$

56. $(c + d)(c - d)(2c - 3d)$

SOLUTION:

Use the FOIL method and then combine like terms.

$$\begin{aligned}(c+d)(c-d)(2c-3d)&=(c^2-d^2)(2c-3d)\\&=2c^3-3c^2d-2cd^2+3d^3\end{aligned}$$

57. $(xy)^2(2xy^2z)^3$

SOLUTION:

Apply the properties of exponents to simplify the expression.

$$\begin{aligned}(xy)^2(2xy^2z)^3&=x^2y^2(8x^3y^6z^3)\\&=8x^5y^8z^3\end{aligned}$$

58. $(3ab^2)^{-2}(2a^2b)^2$

SOLUTION:

$$\begin{aligned}\frac{(2a^2b)^2}{(3ab^2)^2}&=\frac{4a^4b^2}{9a^2b^4} && \text{Power of a Power}\\&=\frac{4a^2}{9b^2} && \text{Quotient of Powers}\end{aligned}$$

59. **LANDSCAPING** Amado wants to plant a garden and surround it with decorative stones. He has enough stones to enclose a rectangular garden with a perimeter of 68 feet, but he wants the garden to cover no more than 240 square feet. What could the width of his garden be?

SOLUTION:

Let x be the length and y be the width of the garden.

$$
\begin{aligned}
2x + 2y &= 68 \\
\Rightarrow x + y &= 34 \\
xy &\le 240 && \text{Area of the garden} \\
(34 - y)y &\le 240 && x = 34 - y \\
34y - y^2 &\le 240 && \text{Distributive Property} \\
y^2 - 34y + 240 &\ge 0 && \text{Subtract 240 from each side.}
\end{aligned}
$$

The solution of the inequality $y^2 - 34y + 240 \ge 0$ is $-\infty < y \le 10$ or $24 \le y < \infty$. Since y represent the width of the garden and the sum of the length and width is 34, the width of the garden is $0 \le y \le 10$ or $24 \le y \le 34$.

Solve each equation by completing the square.

60. $x^2 + 6x + 2 = 0$

SOLUTION:

$$
\begin{aligned}
x^2 + 6x + 2 &= 0 && \text{Original Equation} \\
(x^2 + 6x + 9) - 9 + 2 &= 0 && \left(\frac{6}{2}\right)^2 = 9 \\
(x + 3)^2 - 7 &= 0 && \text{Factor.} \\
(x + 3)^2 &= 7 && \text{Subtract 7 from each side.} \\
x + 3 &= \pm\sqrt{7} && \text{Take the square root of each side.} \\
x &= -3 \pm \sqrt{7} && \text{Subtract 3 from each side.}
\end{aligned}
$$

61. $x^2 - 8x - 3 = 0$

SOLUTION:

$$
\begin{aligned}
x^2 - 8x - 3 &= 0 \\
(x^2 - 8x + 16) - 16 - 3 &= 0 \\
(x - 4)^2 - 19 &= 0 \\
(x - 4) &= \pm\sqrt{19} \\
x &= 4 \pm \sqrt{19}
\end{aligned}
$$

62. $2x^2 + 6x + 5 = 0$

SOLUTION:

$2x^2 + 6x + 5 = 0$	Original equation
$2(x^2 + 3x) + 5 = 0$	Factor GCF from first two terms.
$2\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] + 5 = 0$	Square half of 3.
$2\left(x + \frac{3}{2}\right)^2 - \frac{9}{2} + 5 = 0$	Simplify.
$2\left(x + \frac{3}{2}\right)^2 = -\frac{1}{2}$	Simplify.
$\left(x + \frac{3}{2}\right)^2 = -\frac{1}{4}$	Divide each side by 2.
$x + 1.5 = \pm\sqrt{\frac{-1}{4}}$	Take the square root of each side.
$= \pm\frac{i}{2}$	$\sqrt{-1} = i$
$= -1.5 \pm \frac{i}{2}$	Simplify.

State the consecutive integers between which the zeros of each quadratic function are located.

63.

x	−7	−6	−5	−4	−3	−2	−1	0
$f(x)$	4	1	−3	−8	−1	2	8	16

SOLUTION:

The sign of $f(x)$ changes between the x values −6 and –5, and between –3 and –2. So the zeros of the quadratic function lies between –6 and –5, and between –3 and –2.

64.

x	−2	−1	0	1	2	3	4	5
$f(x)$	−16	−7	−4	3	3	−4	−7	−16

SOLUTION:

The sign of $f(x)$ changes between the x values between 0 and 1, and between 2 and 3. So the zeros of the quadratic function lies between 0 and 1, and between 2 and 3.

65.

x	−2	−1	0	1	2	3	4	5
$f(x)$	6	1	−3	−5	−3	1	6	14

SOLUTION:

The sign of $f(x)$ changes between the x values between –1 and 0, and between 2 and 3. So the zeros of the quadratic function lies between –1 and 0, and between 2 and 3.

66. **BUSINESS** A landscaper can mow a lawn in 30 minutes and perform a small landscape job in 90 minutes. He works at most 10 hours per day, 5 days per week. He earns $35 per lawn and $125 per landscape job. He cannot do more than 3 landscape jobs per day. Find the combination of lawns mowed and completed landscape jobs per week that will maximize income. Then find the maximum income.

SOLUTION:

Let M be the lawns mowed and L be the small landscaping jobs. Since the landscaper will do at most 3 landscaping jobs per day, $L \le 15$. Since he works 10 hours per day, 5 days a week, he can do at most 20 mowing jobs per day or 100 per week. So, $M \le 100$.

If the landscaper does 3 landscaping jobs per day, he can only do 11 mowing jobs per day and work a total of 10 hours per day. So, for the week, if he does 15 landscaping jobs, he can mow 55 lawns.

To maximize the earnings, write a function that relates the number and charge for each type of job. Since he charges $35 for each lawn mowed and $125 for each small landscaping job, $F(M, L) = 35M + 125L$. Next, substitute in the key values of M and L to determine the earnings for each combination of jobs.

(M, L)	**$F(M, L) = 35M + 125L$**	**$F(M, L)$**
(100, 0)	$F(M, L) = 35(100) + 125(0)$	3500
(55, 15)	$F(M, L) = 35(55) + 125(15)$	3800
(0, 15)	$F(M, L) = 35(0) + 125(15)$	1875

15 landscape jobs and 55 lawns; $3800

Find each value if $f(x) = 4x + 3$, $g(x) = -x^2$, and $h(x) = -2x^2 - 2x + 4$.

67. $f(-6)$

SOLUTION:

Substitute –6 for x in $f(x)$.

$$\begin{aligned} f(-6) &= 4(-6) + 3 \\ &= -24 + 3 \\ &= -21 \end{aligned}$$

68. $g(-8)$

SOLUTION:

Substitute –8 for x in $g(x)$.

$$\begin{aligned} g(-8) &= -(-8)^2 \\ &= -64 \end{aligned}$$

69. $h(3)$

SOLUTION:
Substitute 3 for x in $h(x)$.

$$\begin{aligned} h(3) &= -2(3)^2 - 2(3) + 4 \\ &= -2(9) - 6 + 4 \\ &= -18 - 6 + 4 \\ &= -20 \end{aligned}$$

70. $f(c)$

SOLUTION:
Substitute c for x in $f(x)$.

$$\begin{aligned} f(c) &= 4(c) + 3 \\ &= 4c + 3 \end{aligned}$$

71. $g(3d)$

SOLUTION:
Substitute $3d$ for x in $g(x)$.

$$\begin{aligned} g(3d) &= -(3d)^2 \\ &= -9d^2 \end{aligned}$$

72. $h(2b + 1)$

SOLUTION:
Substitute $2b + 1$ for x in $h(x)$.

$$\begin{aligned} h(2b+1) &= -2(2b+1)^2 - 2(2b+1) + 4 \\ &= -2(4b^2 + 4b + 1) - 4b - 2 + 4 \\ &= -8b^2 - 8b - 2 - 4b + 2 \\ &= -8b^2 - 12b \end{aligned}$$